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Distance domination-critical graphs[☆]

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Abstract

A set D of vertices in a connected graph G is called a k-dominating set if every vertex in $G - D$ is within distance k from some vertex of D. The k-domination number of G, $\gamma_k(G)$, is the minimum cardinality over all k-dominating sets of G. A graph G is k-distance domination-critical if $\gamma_k(G - x) < \gamma_k(G)$ for any vertex x in G. This work considers properties of k-distance domination-critical graphs and establishes a best possible upper bound on the diameter of a 2-distance domination-critical graph G, that is, $d(G) \leq 3(\gamma_2 - 1)$ for $\gamma_2 \geq 2$.

Keywords: k-domination number; k-distance domination-critical; Diameter; k-neighborhood

1. Introduction

For the terminology and notation of graph theory not given here, the reader is referred to [1] or [11]. Let $G = (V, E)$ be a connected simple graph. For $S \subseteq V(G)$, $G[S]$ denotes a subgraph of G induced by S. The distance $d_G(x, y)$ between two vertices x and y is the length of a shortest xy-path in G. The diameter of G, $d(G)$, is the maximum distance between any two vertices in G. Let k be a positive integer. For every vertex $x \in V(G)$, the open k-neighborhood $N_k(x)$ of x is defined as $N_k(x) = \{y \in V(G) :\ 1 \leq d_G(x, y) \leq k\}$. The closed k-neighborhood $N_k[x]$ of x in G is defined as $N_k(x) \cup \{x\}$. Let

$$\Delta_k(G) = \max\{|N_k(x)| : \text{for any } x \in V(G)\}.$$

Clearly, $\Delta_1(G) = \Delta(G)$. For a set $X \subset V(G)$, let

$$N_k(X) = \bigcup_{x \in X} N_k(x) \quad \text{and} \quad N_k[X] = \bigcup_{x \in X} N_k[x].$$

A set $D \subset V(G)$ is called a k-dominating set of G if every vertex in $G - D$ is within distance k from some vertex of D. The minimum cardinality over all k-dominating sets of G is called the k-domination number of G and is

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denoted by $\gamma_k(G)$. A minimum k-dominating set is called a γ_k-set for short. The concept of the k-dominating set was introduced by Chang and Nemhauser [5,6] and could find applications for many situations and structures which give rise to graphs; see the books by Haynes et al. [2,3].

Brigham et al. [4] define a vertex v of a graph G as being critical if $\gamma(G-v) < \gamma(G)$. The graph G is vertex domination-critical (or γ-critical) if each vertex is critical, which has been extensively studied (see, for example, [4,7–9]). For $k \geq 1$, a vertex v is k-distance domination-critical if $\gamma_k(G-v) < \gamma_k(G)$ and G is k-distance domination-critical, γ_k-critical for short, if each vertex in G is k-distance domination-critical, which was studied by Henning et al. [10].

Fulman et al. [8] showed that a γ-critical graph G is regular if its order is $(\Delta+1)(\gamma-1)+1$, and its diameter $d \leq 2(\gamma-1)$ if $\gamma \geq 2$. In this work, we show that for a γ_k-critical graph G, $|N_k(x)| = \Delta_k$ for any $x \in V(G)$ if its order is $(\Delta_k+1)(\gamma_k-1)+1$, and its diameter $d \leq 2k(\gamma_k-1)$. In particular, for $k=2$, we have $d \leq 3(\gamma_2-1)$ if $\gamma_2 \geq 2$. Clearly, our results generalize ones of Fulman et al.

2. Some lemmas

In what follows, for any a vertex v in G, we use D_v to denote a minimum k-dominating set of the subgraph $G_v = G - v$, and D_v^u to denote the set $D_v \cup \{u\}$ for $u \in V(G)$.

Lemma 2.1. *If G is a connected γ_k-critical graph, then $\gamma_k(G-v) = \gamma_k(G) - 1$ for any $v \in V(G)$.*

Proof. Let G be a γ_k-critical graph. Then, it is clear that $\gamma_k(G-v) \leq \gamma_k(G)-1$ for any $v \in V(G)$. But if there exists a vertex $u \in V(G)$ such that $\gamma_k(G-u) < \gamma_k(G)-1$, then D_u^u is a k-dominating set of G with cardinality less than $\gamma_k(G)$, a contradiction. Thus, $\gamma_k(G-v) = \gamma_k(G)-1$ for any $v \in V(G)$. ∎

Let k be a positive integer. The k-th power of a graph G is the graph G^k with vertex set $V(G^k) = V(G)$ and edge set $E(G^k) = \{xy : 1 \leq d_G(x,y) \leq k\}$. The following lemma holds directly from the definition of G^k.

Lemma 2.2. *$\Delta(G^k) = \Delta_k(G)$ and $\gamma(G^k) = \gamma_k(G)$ for any graph G and each $k \geq 1$.*

Lemma 2.3 (*G. MacGillvray*). *For each $k \geq 1$, a graph G is $\gamma_k(G)$-critical if and only if G^k is $\gamma(G^k)$-critical.*

Proof. This is clear for $k=1$, so we assume $k \geq 2$ below.

Suppose that G is a γ_k-critical graph. Let $x \in V(G)$. By the Lemma 2.2, a k-dominating set of $G-x$ is a dominating set of $(G-x)^k$. Since $(G-x)^k$ is a spanning subgraph of $G^k - x$, then it follows that G^k is $\gamma(G^k)$-critical.

For the converse, suppose that G^k is $\gamma(G^k)$-critical. Then there must exist a dominating set D of $G^k - x$ such that D contains no vertex y such that $d_G(x,y) \leq k$. Therefore, no edge of G^k joining a vertex of D to a vertex of $V(G^k) - (D \cup \{x\})$ arises in G^k from a path of length at most k that contains x. It follows that D is a dominating set of $(G-x)^k$, and hence a k-dominating set of $G-x$. This completes the proof. ∎

Remarks. Lemma 2.3 and its proof are due to G. MacGillvray [unpublished].

Lemma 2.4. *For each $k \geq 1$, if the vertices x and y are two vertices in G such that $d_G(x,y) = d(G)$, then $d_{G^k}(x,y) = d(G^k)$. Furthermore, $d(G^k) = \lceil \frac{d(G)}{k} \rceil$.*

Proof. Suppose x and y are two vertices in G such that $d_G(x,y) = d(G)$. If $d_{G^k}(x,y) < d(G^k)$, then there must exist two vertices x' and y' such that $d_{G^k}(x',y') = d(G^k)$. By the definition of G^k, we get a contradiction for $d_G(x',y') > d_G(x,y) = d(G)$.

Let $d(G) = mk + t$, where $0 \leq t < k$. For $t = 0$, we have $d(G^k) = m = \frac{d(G)}{k}$ by the definition of G^k. For $t \neq 0$, let x and y be two vertices in G such that $d_G(x,y) = d(G)$, and we consider an xy-path of length $d(G)$. Then there must exist a vertex v on this xy-path such that $d_G(x,v) = mk$ and $d_G(v,y) = t$. By the definition of G^k, we have $d_{G^k}(x,v) = m$ and $d_{G^k}(v,y) = 1$. Therefore, $d(G^k) = d_{G^k}(x,y) = d_{G^k}(x,v) + d_{G^k}(v,y) = m+1 = \lceil \frac{d(G)}{k} \rceil$. ∎

Lemma 2.5 (*Fulman et al. [8]*). *If G is a γ-critical graph with order n, then $d_G(x) = \Delta(G)$ for any $x \in V(G)$ if $n = (\Delta+1)(\gamma-1)+1$, and its diameter $d \leq 2(\gamma-1)$ if $\gamma \geq 2$.*

3. Main results

Theorem 3.1. *Let G be a connected γ_k-critical graph and $v \in V(G)$; then there are two vertices x and y in $N_k(v)$ such that $d_G(x, y) > k$.*

Proof. We only need to show that G must not be γ_k-critical if $d_G(x, y) \le k$ for any two vertices x and y in $N_k(v)$. Suppose on the contrary that G is γ_k-critical. Take $x \in N_k(v)$ and consider the subgraph G_x. Since any γ_k-set D_x of G_x must include a vertex, say y, in $N_k[v]$, then D_x must also k-dominate x since $d_G(x, y) \le k$. Thus, D_x is also a k-dominating set of G with cardinality less than $\gamma_k(G)$, which contradicts the definition of $\gamma_k(G)$. ■

Theorem 3.2. *Let G be a γ_k-critical graph of order n. Then $n \le (\Delta_k(G) + 1)(\gamma_k(G) - 1) + 1$. Moreover, if the equality holds then $|N_k(x)| = \Delta_k(G)$ for any $x \in V(G)$.*

Proof. Let v be a vertex of G. Since G is a γ_k-critical graph of order n, $|D_v| = \gamma_k(G) - 1$ by Lemma 2.1. Since each vertex of D_v can k-dominate at most $(\Delta_k(G) + 1)$ vertices, then D_v can k-dominate at most $(\Delta_k(G) + 1)(\gamma_k(G) - 1)$ vertices, which implies that

$$n = |V(G_v)| + 1 \le (\Delta_k(G) + 1)(\gamma_k(G) - 1) + 1.$$

We now assume $n = |V(G_v)| + 1 = (\Delta_k(G) + 1)(\gamma_k(G) - 1) + 1$. By Lemma 2.2, we have $\gamma(G^k) = \gamma_k(G)$. By Lemma 2.3, we have G^k is $\gamma(G^k)$-critical graph. By Lemma 2.5, we have $|d_{G^k}(x)| = \Delta(G^k)$ for any $x \in V(G)$. By the definition of G^k, $|N_k(x)| = |d_{G^k}(x)| = \Delta(G^k) = \Delta_k(G)$ for any $x \in V(G)$. ■

Theorem 3.3. *Let G be a γ_k-critical graph. Then its diameter $d(G) \le 2k(\gamma_k - 1)$ if $\gamma_k \ge 2$.*

Proof. By Lemmas 2.2–2.5, we have $\frac{d(G)}{k} \le d(G^k) \le 2(\gamma(G^k) - 1)$. So we get the theorem. ■

By Theorem 3.3, we have $d(G) \le 4(\gamma_2 - 1)$ for $k = 2$. However, we can get a better upper bound than Theorem 3.3 and this bound is tight.

Theorem 3.4. *Let G be a γ_2-critical graph. If $\gamma_2 \ge 2$, then the diameter G*

$$d(G) \le 3(\gamma_2 - 1),$$

and this bound is best possible.

Proof. Let x and y be two vertices in G such that $d_G(x, y) = d$. Define $X_j = \{z \in V(G) : d_G(x, z) = j\}$ and $U_j = X_0 \cup X_1 \cup \cdots \cup X_j$, where $0 \le j \le d$.

Let D be a γ_2-set of G. For $j > 1$, the subgraph $G[U_j]$ is said to be D-full if it satisfies that $j \le 3(|D \cap U_j| - 1)$. It is easy to check that $G[U_3]$ is D_x^x-full.

If $G[U_d]$ is D-full for some γ_2-set D, then $d \le 3(|D \cap U_d| - 1) = 3(\gamma_2 - 1)$, and so the theorem follows since $G = G[U_d]$. Suppose that $G[U_d]$ is not D-full for any γ_2-set D below. Since $G[U_3]$ is D_x^x-full, there must exist an integer $h < d$ such that for any γ_2-set D, $G[U_i]$ is not D-full for any $i > h$, but there exists a γ_2-set D such that $G[U_h]$ is D-full. Let D be such a γ_2-set below.

Now we have $h \le 3(|D \cap U_h| - 1)$, $h + 1 > 3(|D \cap U_{h+1}| - 1)$ and $h + 2 > 3(|D \cap U_{h+2}| - 1)$. Then $|D \cap U_h| \ge 1 + \frac{h}{3}$, $|D \cap U_{h+1}| < 1 + \frac{h+1}{3}$ and $|D \cap U_{h+2}| < 1 + \frac{h+2}{3}$.

Let $h = 3m + r$ for some integer m, where $0 \le r \le 2$. If $1 \le r \le 2$, then $|D \cap U_h| \ge 1 + \lceil \frac{3m+r}{3} \rceil = 2 + m$, while $|D \cap U_{h+1}| < 1 + \lceil \frac{3m+r+1}{3} \rceil = 2 + m$, so they contradict each other. Therefore, $h = 3m$, $|D \cap U_h| \ge 1 + m$ while $|D \cap U_{h+1}| < 1 + \lceil \frac{3m+1}{3} \rceil$ and $|D \cap U_{h+2}| < 1 + \lceil \frac{3m+2}{3} \rceil$. Thus we have $|D \cap U_h| = 1 + m$, $D \cap X_{h+1} = \emptyset$ and $D \cap X_{h+2} = \emptyset$.

Suppose that $d > h + 2$. If $D \cap X_{h+3} \ne \emptyset$, then $|D \cap U_{h+3}| \ge 1 + (1 + m) = 2 + m = 1 + \frac{h+3}{3}$, contradicting the maximality of h, so $D \cap X_{h+3} = \emptyset$. Hence we have $d > h + 3$. But if $|D \cap X_{h+4}| \ge 2$, we have $|D \cap U_{h+4}| \ge 2 + (1 + m) = 3 + m > 1 + \frac{h+4}{3}$, again contradicting the maximality of h. So $|D \cap X_{h+4}| \le 1$.

Case 1. $|D \cap X_{h+4}| = 1$, that is, there is exactly one vertex of D, say u, in X_{h+4}.

We first claim that the vertex u must 2-dominate X_{h+3}. Otherwise, there exists at least one vertex of the 2-dominating set D in X_{h+5}, that is $D \cap X_{h+5} \ne \emptyset$. Then $|D \cap U_{h+5}| \ge 1 + (2 + m) = 3 + m > 1 + \frac{h+5}{3}$, contradicting the maximality of h; then $D \cap X_{h+5} = \emptyset$. If u could not 2-dominate X_{h+4}, then there must exist at least one vertex in X_{h+5} or X_{h+6}, that is, $D \cap X_{h+5} \neq \emptyset$ or $D \cap X_{h+6} \neq \emptyset$; thus $|D \cap U_{h+5}| \geq 1 + (2 + m) = 3 + m = 1 + \frac{h+5}{3}$ or $|D \cap U_{h+6}| \geq 1 + (2 + m) = 3 + m = 1 + \frac{h+6}{3}$, contradicting the maximality of h. So u could 2-dominate $X_{h+3} \cup X_{h+4}$.

Now consider G_u and the minimum 2-dominating set D_u of G_u. Then $D_u \cap (X_{h+3} \cup X_{h+4}) = \emptyset$; otherwise, D_u also could 2-dominate u. Assume that $|D_u \cap U_{h+2}| \geq 1 + m$ and take a vertex $w \in V(X_{h+3})$; then D_u^w could 2-dominate G, and $|D_u^w \cap U_{h+3}| \geq 2 + m = 1 + \frac{h+3}{3}$, contradicting the maximality of h. Thus we have $|D_u \cap U_{h+2}| \leq m$. Noticing that $D_u \cap (X_{h+3} \cup X_{h+4}) = \emptyset$, then $|D_u \cap U_{h+4}| \leq m$.

Let $D' = (D - U_{h+2}) \cup (D_u \cap U_{h+4})$. We claim that D' is a 2-dominating set of G, since $G - U_{h+2}$ must be able to be 2-dominated by $D - U_{h+2}$, and the vertices in U_{h+2} must be able to be 2-dominated by $D_u \cap U_{h+4}$.

But $|D'| \leq (\gamma_2(G) - 1 - m) + m \leq \gamma_2(G) - 1$, a contradiction to the minimality of $\gamma_2(G)$. So $|D \cap X_{h+4}| \neq 1$.

Case 2. $|D \cap X_{h+4}| = 0$; then we have $d > h + 4$.

Since D is a 2-dominating set of G, then $D \cap X_{h+5} \neq \emptyset$. But if $|D \cap X_{h+5}| \geq 2$, then $|D \cap U_{h+5}| \geq 2 + (1 + m)$ $= 3 + m > 1 + \frac{h+5}{3}$, a contradiction to the maximality of h. Then $|D \cap X_{h+5}| = 1$, that is there is exactly one vertex u in X_{h+5} and u could 2-dominate the vertices in X_{h+3}. But if u could not 2-dominate all the vertices in X_{h+4}, then $D \cap X_{h+6} \neq \emptyset$. If $D \cap X_{h+6} \geq 1$, then $|D \cap U_{h+6}| \geq 1 + (2 + m) = 3 + m = 1 + \frac{h+6}{3}$, a contradiction to the maximality of h. Then $|D \cap X_{h+6}| = \emptyset$, and u also 2-dominates $X_{h+3} \cup X_{h+4}$.

Now consider G_u and the minimum 2-dominating set D_u of G_u. Then $D_u \cap (X_{h+3} \cup X_{h+4}) = \emptyset$; otherwise, D_u also could 2-dominate u. Assume that $|D_u \cap U_{h+2}| \geq 1 + m$. Take a vertex $w \in V(X_{h+3})$; then D_u^w could 2-dominate G, and $|D_u^w \cap U_{h+3}| \geq 2 + m = 1 + \frac{h+3}{3}$, contradicting the maximality of h. Thus we have $|D_u \cap U_{h+2}| \leq m$. Noticing that $D_u \cap (X_{h+3} \cup X_{h+4}) = \emptyset$, then $|D_u \cap U_{h+4}| \leq m$.

Let $D' = (D - U_{h+2}) \cup (D_u \cap U_{h+4})$. We claim that D' is a 2-dominating set of G, since $G - U_{h+2}$ must be able to be 2-dominated by $D - U_{h+2}$, and the vertices in U_{h+2} must be able to be 2-dominated by $D_u \cap U_{h+4}$.

But $|D'| \leq (\gamma_2(G) - 1 - m) + m \leq \gamma_2(G) - 1$, a contradiction to the minimality of $\gamma_2(G)$. Then $|D \cap X_{h+4}| \neq 0$.

Now we have that $h < d \leq h + 2$, that is, $d = 3m + 1$ or $d = 3m + 2$; otherwise $d \leq 3(\gamma_2 - 1)$. This implies that the theorem is true for any graph whose diameter is a multiple of 3.

Now assume that $d = 3m + 1$ or $d = 3m + 2$. Let G_1, G_2 and G_3 be three vertex disjoint copies of G. And let x_1 be an end-vertex of the diameter of the graph G_1, let x_2 and x_2' be two end-vertices of the diameter of the graph G_2 such that $d_{G_2}(x_2, x_2') = d$, and x_3 be an end-vertex of the diameter of the graph G_3. And let $\{a_1, a_2\}$, $\{b_1, b_2\}$, $\{c_1, c_2\}$ and $\{d_1, d_2\}$ be four edges. Let H be the graph constructed from the disjoint union $G_1 \cup G_2 \cup G_3 \cup \{a_1, a_2\} \cup \{b_1, b_2\} \cup \{c_1, c_2\} \cup \{d_1, d_2\}$ by adding the edges x_1a_1, x_1b_1, x_2a_2, x_2b_2, $x_2'c_1$,$x_2'd_1$, x_3c_2 and x_3d_2. Then the diameter of H is $3d + 6$, which is a multiple of 3.

We claim that H is $3\gamma_2$-critical. First we show that $\gamma_2(H) = 3\gamma_2(G)$. Because $D_{x_1}^{x_1} \cup D_{x_2}^{x_2} \cup D_{x_3}^{x_3}$ is a 2-dominating set of H, then $\gamma_2(H) \leq 3\gamma_2(G)$. Suppose that $\gamma_2(H) < 3\gamma_2(G)$ and assume that D_H is a minimum 2-dominating set of H.

(a) If $D_H \cap \{a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2\} = \emptyset$, then some G_i, where $i \in \{1, 2, 3\}$, could be 2-dominated by some vertex set with cardinality less than $\gamma_2(G)$ vertices. This is a contradiction to the minimality of $\gamma_2(G)$.

(b) If $D_H \cap \{a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2\} \neq \emptyset$, and since $\{a_1, a_2, b_1, b_2\}$ and $\{c_1, c_2, d_1, d_2\}$ are all to be 2-dominated by D_H, then we must have $|D_H \cap \{x_1, x_2, a_1, a_2, b_1, b_2\}| \geq 2$ and $|D_H \cap \{x_2', x_3, c_1, c_2, d_1, d_2\}| \geq 2$. And by the minimality of D_H, we have $|D_H \cap \{x_1, x_2, a_1, a_2, b_1, b_2\}| = 2$ and $|D_H \cap \{x_2', x_3, c_1, c_2, d_1, d_2\}| = 2$. Then we construct a new set $D_H' = (D_H - \{a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2\}) \cup \{x_1, x_2, x_2', x_3\}$, and D_H' is also a 2-dominating set of H such that $D_H' \cap \{a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2\} = \emptyset$. By (a), we also get a contradiction.

It follows from (a) and (b) that we have $\gamma_2(H) = 3\gamma_2(G)$.

In what follows, we show that H is $3\gamma_2$-critical.

(a′) If a vertex is removed from a copy of G in H, say a vertex y in G_1, then $D_y \cup D_{x_2}^{x_2} \cup D_{x_3}^{x_3}$ is a 2-dominating set of $H - \{y\}$ with cardinality $3\gamma_2(G) - 1$.

(b′) If a vertex in $\{a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2\}$ is removed from H, say a_1, then $D_{x_1} \cup D_{x_2} \cup D_{x_3}^{x_3} \cup \{b_2\}$ is a 2-dominating set of $H - \{a_1\}$ with cardinality $3\gamma_2(G) - 1$.

It follows from (a′) and (b′) that we have that H is $3\gamma_2$-critical. And $d(H) = 3d + 6$.

Since the theorem is true for all graphs whose diameter is a multiple of 3, then $3d + 6 \leq 3(3\gamma_2 - 1)$, which implies that $d \leq 3(\gamma_2 - 1)$ as desired.

In order to complete the proof of the theorem, we show that this bound is best possible.

Let G be a path on n vertices, denoted by $\{u_1, u_2, \ldots, u_n\}$. Replacing each edge by two internally disjoint paths of length 3, then for the resulting graph H it is easily verified that H is an n-critical graph with diameter $d(H) = 3(n-1)$. Then the proof of the theorem is completed. ■

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